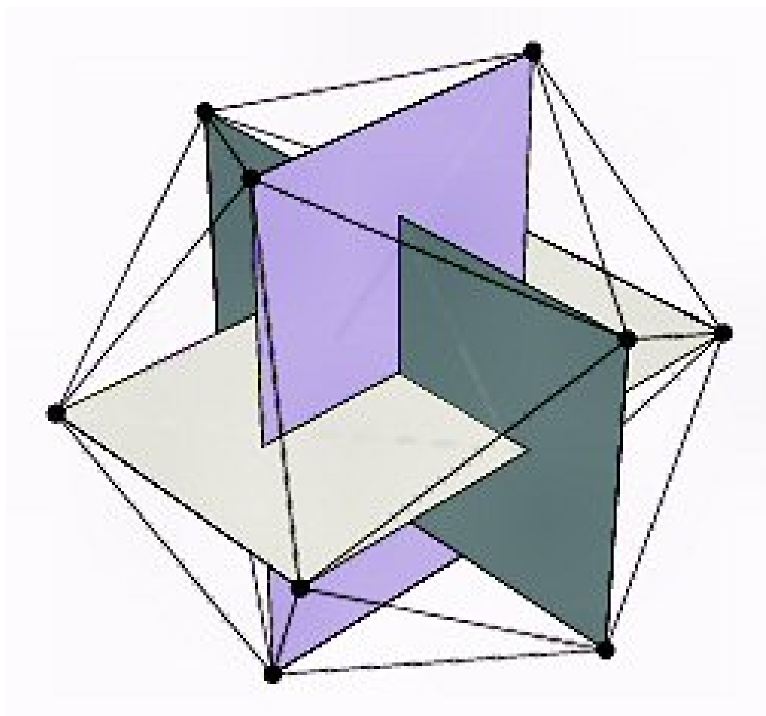
MATHEMATICS

and

ARCHITECTURAL DESIGN



CURRICULUM DEVELOPMENT

SECTION 6.0

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SK85ON23

INDEX:

ABSTRACT:

Mathematics, a technical science, plays an integral role in architectural design. The use of mathematics is applied both artistically and practically in creating a design solution.

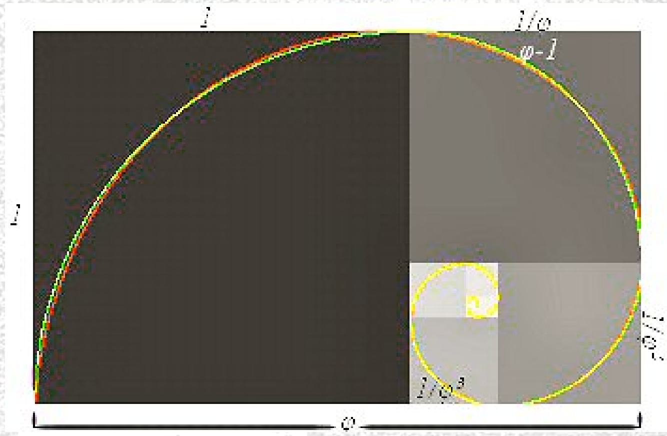


Figure 1: Fibonacci Algorithm

PREAMBLE:

The function of mathematics as an element of architectural design is two-fold. The first function serves as the economic factors relative to a proposed design solution. This function uses the proposed size (floor area and heights), material elements and developmental requirements. These items are combined with mathematical formulae to create a budget for the construction as well as for future operations and maintenance costs. This function was referenced in the original proposal as the 'Building and Area Calculations' component of mathematics.

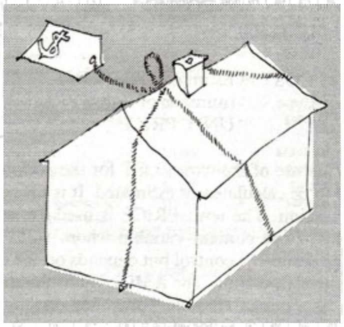


Figure 2: Total Building Cost

The second function serves as a principal component of the design rationale. The size, proportion and area distribution of the design spaces are mathematically derived based on the theory of relationships. This function is referred to in the proposal document as the 'Proportional Geometry' component of mathematics.

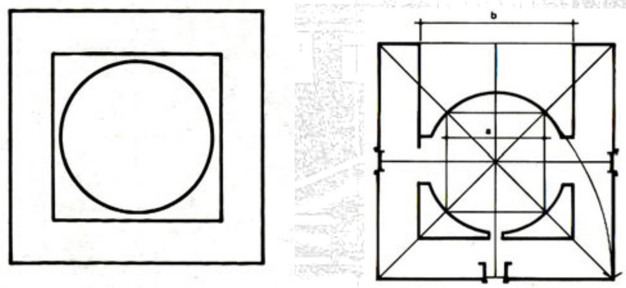


Figure 3: Stockholm Civic Library Parti and Geometry

Construction is an expensive venture. The mathematics of the process plays an important role whether the design is a renovation or expansion to an existing building or if new construction is considered. Size and scale relative to the end user affects the atmosphere and perception of a space. Size and scale relative to construction and maintenance/operations costs affects the potential viability of achieving a cost effective design.

Mathematics is one component of the architectural design process that remains behind the scenes. The only time it is "visible" happens in bad news (over budget) or bad solutions (under-sized). Consideration and application of mathematic principles early in the design process will aid in fostering potential success of the project.

COMPONENT INITIATIVE:

The goal of this section is to provide an understanding of how mathematics applies to the architectural design process. Students will be better able to understand the considerations and implications of design decisions through a means of mathematical analysis. This analysis type provides clear and concise answers to questions of benefit and cost. These answers are black & white, as mathematics is a technical science, leaving no room for “interpretation” in the way artistic or social implications may provide.

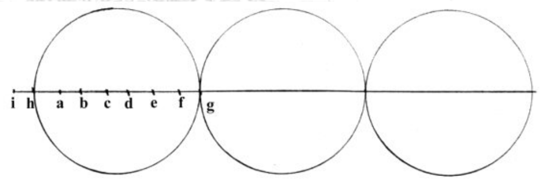


Figure 4: Circular Division

Mathematics plays a role in our lives every day. It plays an equally important role in the architectural and construction industries; “time is money”. The study of materials provided in this section will aid students in developing their understanding of mathematical principles related to design (theory) and construction (technical).

COMPONENT COURSE MATERIALS:

The component course materials for this section are divided into two main parts – technical applications and theoretical applications. These two parts illustrate the extent through which mathematics plays an integral role in the architectural design process by the manner in which they are implemented.

Technical elements represents the hard facts of design containing fixed or determined room data (area and volume) and budget calculations (using area, volume, material types and known or historical unit rates) to analyze the potential solution.

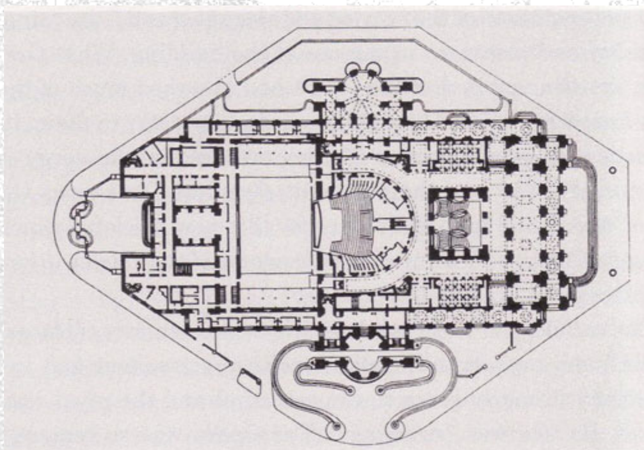


Figure 5: Paris Opera House

Theoretical applications incorporate the geometry and proportional relationships of spaces and features within a design solution. The theoretical applications are used as a means to develop the spatial area relationships, massing of the solution and proportion of parts to the whole. The two main parts work in concert through a circular process that forms an integral component of the design process.

The course section outlines the applications and various uses of these two parts, complete with illustrative examples. Visual application is included where applicable, in an effort to clearly explain the concept.

Figure 6: Budget Breakdown

A portion of this course is used to explain the overall development process as it relates to full-scale architectural development. This portion is included to provide a total view of the complex considerations that apply to the design and development process. This portion is related to the 'Mathematics' curriculum in the manner by which the design solution is often analyzed "by the numbers", as opposed to analyzing a solution through the degree that it satisfies the user's needs and provides for an enhanced environment.

Instructional Strategy

- **Direct Instruction**
  - Lecture series with written material hand-outs.
  - Visual presentation of geometric analysis and mathematical derivations.
- **Indirect Instruction**
  - Textbook reading and assigned problems.
  - Study of mathematic formulas and applications.
- **Independent Study**
  - Student work on homework assignments.
  - Geometric analysis of completed or proposed works.
- **Interactive Instruction**
  - Demonstration and resolution of geometric principles.
  - Demonstration of proportional relationships.

Student Activities

- **Oral**
  - Presentation on mathematic influences.
  - Discussion on proportion and geometry in architectural design.
- **Visual**
  - Graphic representation of geometric and proportional analysis.
- **Kinesthetic**
  - Activities involving measurement tools to determine area, volume and quantity.
  - Proportional model making including existing and proposed solutions.
- **Written**
  - Mathematical problem solving worksheets.
  - Report preparation on geometric and proportional analysis.

Assessment Method

- **Pencil and Paper Method**
  - Written testing on mathematical principles.
  - Report submission on geometric and proportional analysis.
  - Graphic submission in design analysis.
- **Performance Assessments**
  - Participation in class activities.
  - Presentation assessment relative to report and analysis studies.
  - Completion of assigned tasks.
- **Personal Assessments**
  - Understanding of mathematical applications.
  - Enhanced knowledge base relative to geometric concepts.
  - Increased understanding of proportional relationships.

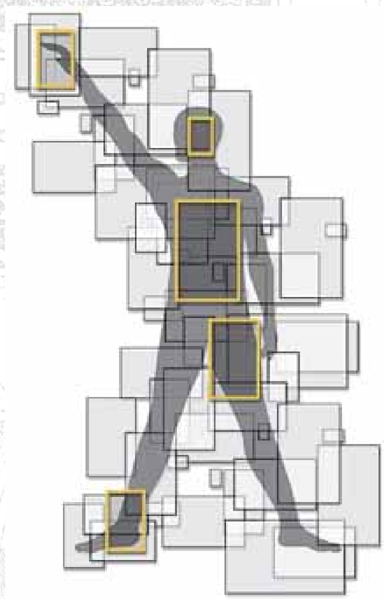


Figure 7: Human Figure Ratio

Common Essential Learning

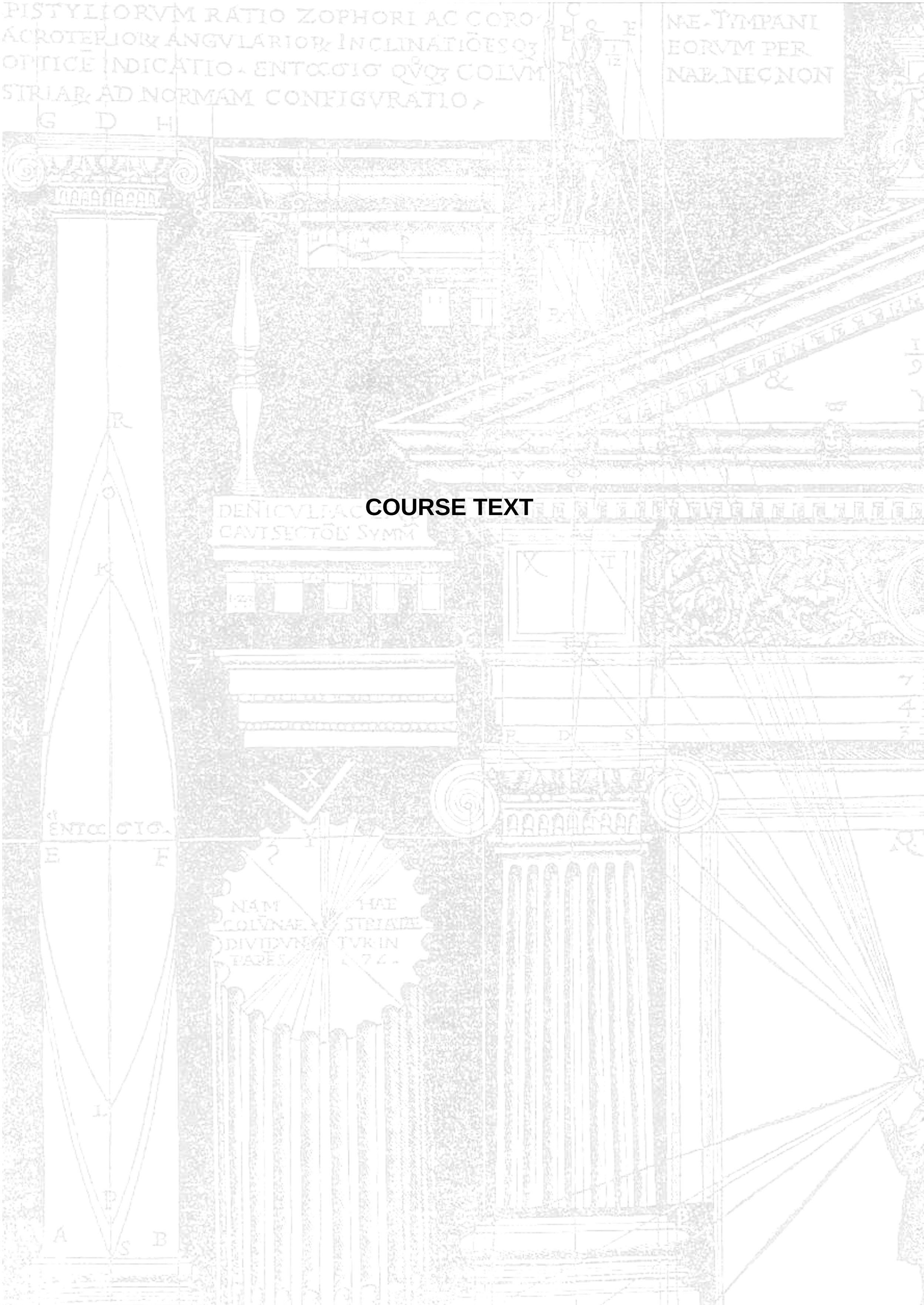
- **Communication**
  - Communication techniques relative to graphic analysis.
  - New terminology and definitions.
- **Creative and Critical Thinking**
  - Ability to perceive and apply geometric and proportional theory.
  - Understanding of the mathematical complexity within the design process.
- **Independent Learning**
  - Research, assignment, and written submissions.
  - Independent study of established design concepts and applications.
- **Numeracy**
  - Mathematical calculations for area, volume, quantity and proportion.
  - Application of formula relating to design estimating.
  - Application of geometric principles.
  - Integration of percentages within mathematical calculation; applications of budget variances and the influences of costs (hard and soft) related to design calculations.
  - Creation of spreadsheets to provide realistic analysis of budgets and design area breakdown.
- **Technological Literacy**
  - Understanding of budget estimates.
  - Understanding of development costs and influences on design solutions.
  - Understanding of material types, costs, and impact on design solution.
  - Understanding of long-term effects relative to design decisions.
- **Personal Social Values and Skills**
  - Group project activities.
  - Understanding of group participation relative to design solutions in a construction environment.

Environment

- **Classroom Climate**
  - Visual access for lecture and presentation.
  - Open area for graphic analysis.
- **Physical Setting**
  - Classroom/lecture style setting.
  - Lab setting for graphic production.
  - Technology zone for computer access.
- **Flexible Student Groupings**
  - Student group projects for proportion and geometric study.
  - Group interaction for mathematical analysis and role playing (site conditions).
- **Extensions Beyond Classroom Settings**
  - Outside analysis of existing buildings: photography and measurement.
  - Written Assignments relative to personal residential environments.
  - Site tours through existing facilities.
- **Community Experiences**
  - Site trips to developed areas for analysis.
  - Interaction with building owners to conduct on-site review.

Materials / Resources Required

- **In-Room Supplies**
  - audio-visual resources
  - graphic production materials
  - computer stations
  - measurement tools
  - photograph tools
- **External supplies**
  - building photographs
  - research materials
  - access to existing site locations

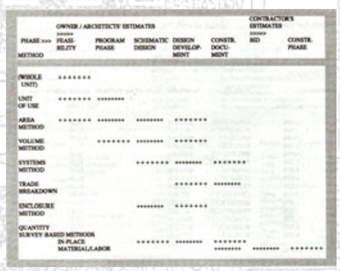


COURSE TEXT

INTRODUCTION

Technical applications of mathematics involve the facets of design and construction. It is important to know the scope of a project at the outset. This information is received from the client in the form of building area (square footage), building components (10-suite apartment building), or budget allowed (definition of total funds available). This information typically arrives in response to the architect's questions of:

- What are we doing? (building type)
- Who is it to serve? (occupant / function)
- Where is it to be located? (site area)
- When is it to be constructed? (seasonal criteria)
- How large is the budget? (total funds available)
- Why is this project initiated? (reasons for development)



PHASE >>> METHOD	OWNER / ARCHITECTS' ESTIMATES >>>>> FEASI-BILITY	PROGRAM PHASE	SCHEMATIC DESIGN	DESIGN DEVELOP-MENT	CONSTR. DOCU-MENT	CONTRACTOR'S ESTIMATES >>>>> BID	CONSTR. PHASE
(WHOLE UNIT)	+++++++						
UNIT OF USE	+++++++	+++++++					
AREA METHOD	+++++++	+++++++	+++++++	+++++++			
VOLUME METHOD		+++++++	+++++++	+++++++			
SYSTEMS METHOD			+++++++	+++++++	+++++++		
TRADE BREAKDOWN				+++++++	+++++++		
ENCLOSURE METHOD			+++++++	+++++++			
QUANTITY SURVEY BASED METHODS IN-PLACE			+++++++	+++++++	+++++++		
MATERIAL/LABOR					+++++++	+++++++	+++++++

Figure 8: Cost Estimate Stages

Mathematics is applied at the outset of a project to determine the initial feasibility of success. The first application comes through using historical data (cost per square metre) multiplied by the client's desired building size.

A simple formula of:

- cost per square metre = unit rate (UR)
- proposed building area = square metres (SM)
- UR x SM = estimated construction budget

This process can be applied in any arrangement, so long as two of the three components are known. Architectural design skills provide the experience of known rates for construction in certain areas (rural or urban) and building types (garage to high-rise structures). This knowledge forms one part of the basic equation, used with the client's data to evaluate the design opportunities.

The information provided through this first calculation will be an early indicator of the nature of the project. If the historical building type cost is $2,000 per square metre, but the calculation reveals an allowable rate of $1,000 per square metre, then the discussion must ensue to either increase the budget or decrease the proposed size, or vary the finishes of the proposed solution. An example of the impact of finishes follows elsewhere in this section.

The functions of mathematics, as related to architectural design, utilize the gross and net areas of a design solution.

(1) Gross Area: refers to the total footprint of an area, building or site. This area includes all walls and projections.

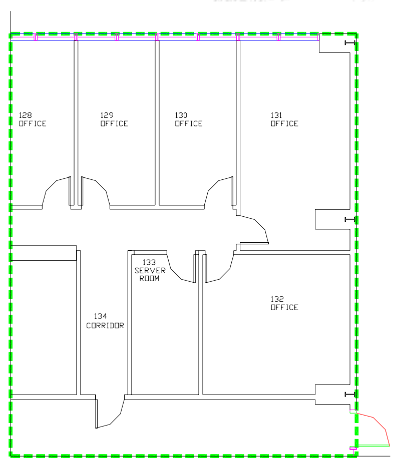


Figure 9: Gross Floor Area Outline

(2) Net Area: refers to the usable or available area that can be occupied. This area does not include walls, projections or surface areas that do not form part of the floor surface.

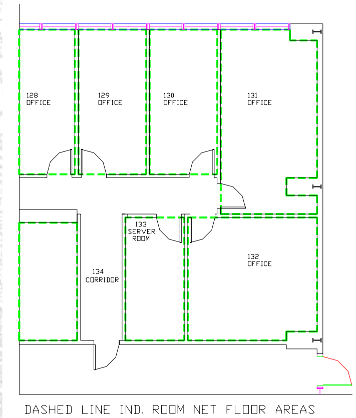


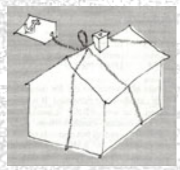
Figure 10: Net Floor Area Outline

The calculation to provide net and gross areas is once again simple math (width x length = floor area). This calculation becomes slightly more complex in the case of unique floor plates and complex areas where it may be required to break the area down into easily identified geometric sections. Computers are a very useful tool in providing floor areas for use in this type of calculation.

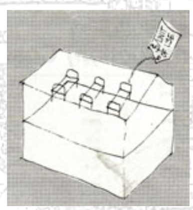
COST ESTIMATING

Bulk estimating can be achieved at the outset of a project in a variety of methods. Each of these methods uses established rates (unit cost) which have been calculated based on past projects. Examples of the various methods include [1]

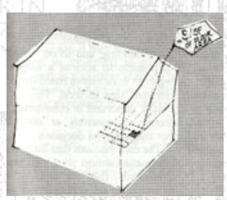
.1 Whole Unit Method: costs quoted for an entire unit (house or garage) based on known costs of similar items (i.e. a one-car garage costs $6,500).



.2 Unit-of-Use Method: costs quoted using historical data on similar facilities (the cost of a school calculated per desk, cost of a hospital calculated per bed). The unit price in this case allows for the costs associated with all the additional service spaces, using the single unit (desk or bed) as a basis of comparison between like building types.

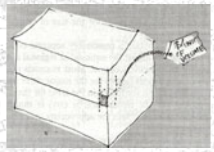


.3 Area Method: costs quote on the basis of square metres, based on building type. This method was discussed earlier in this section.

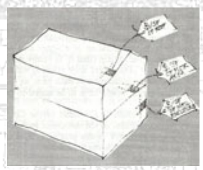


[1] Building Economics For Architects, P. 16

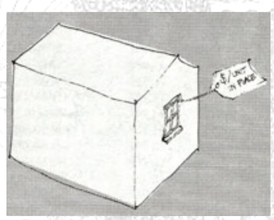
.4 Volume Method: costs quoted on the basis of building volume in cubic metres. This method allows for the height of a space as well as the floor area. This method is most commonly used in Europe.



.5 Enclosure Method: costs calculated in an individual basis for floors, walls, and roofs. This method begins to isolate the design components and uses individual unit rates for each component. A higher level of accuracy is possible in this method than previous methods, as more detail is involved in analyzing the design solution.



.6 Systems Method: costs are calculated in a manner similar to the area method but defined in greater detail. This method isolates each sub-system of the building. This method relates to the systems of composite construction (the total floor or wall system, not the individual pieces).



.7 Trade Breakdown Method: costs are calculated on the basis of the individual construction trades involved in a construction project. The breakdown in this category is found through the required trade work, not the composite sub-systems as noted above.

.8 Quantity Survey Method ('Elemental Breakdown' method): costs are calculated on the actual quantity of each individual material with installation costs added to arrive at a detailed calculation for the entire design. This method involves the most work, requires the most detailed information and provides the most exact calculation. This method is often used immediately before one calls for contractors' quotations as a final check that the project remains on budget.

Each of these methods can be used to evaluate potential design alternatives. The more detailed the method, the more accurate the budget estimate will be.

Each of these methods can be refined and adjusted to suit the specific design type. Factors that will impact an estimate include:

.1 Building Type: warehouse, residential, office, etc. This factor will determine the unit rate (cost per square metre) that is applied in the selected method.

.2 Size: the overall size or scale of a project will have an impact on the unit rate applied to a project. There is a factor known as "economy of scale" that applies once a project exceeds a certain size (i.e. the cost of installing a single light switch in your home is much higher as a "cost per switch" unit rate than would apply per switch if 1,000 light switches were added to a construction project).

.3 Location: the location of a project (rural or urban, developed or undeveloped site, downtown or suburb) will affect unit rates due to potential additional work, travel, sustenance and lodging and shipping costs that may apply.

.4 <u>Building Height</u>: as buildings increase in height, there will be a corresponding increase in unit rate costs. This increase is due to the operational magnitude of materials movement, travel and required equipment. These items must be weighed against the possible reduction in cost seen in the "economy of scale" unit rate, where a greater number of devices or area may result in a lower cost per square metre.

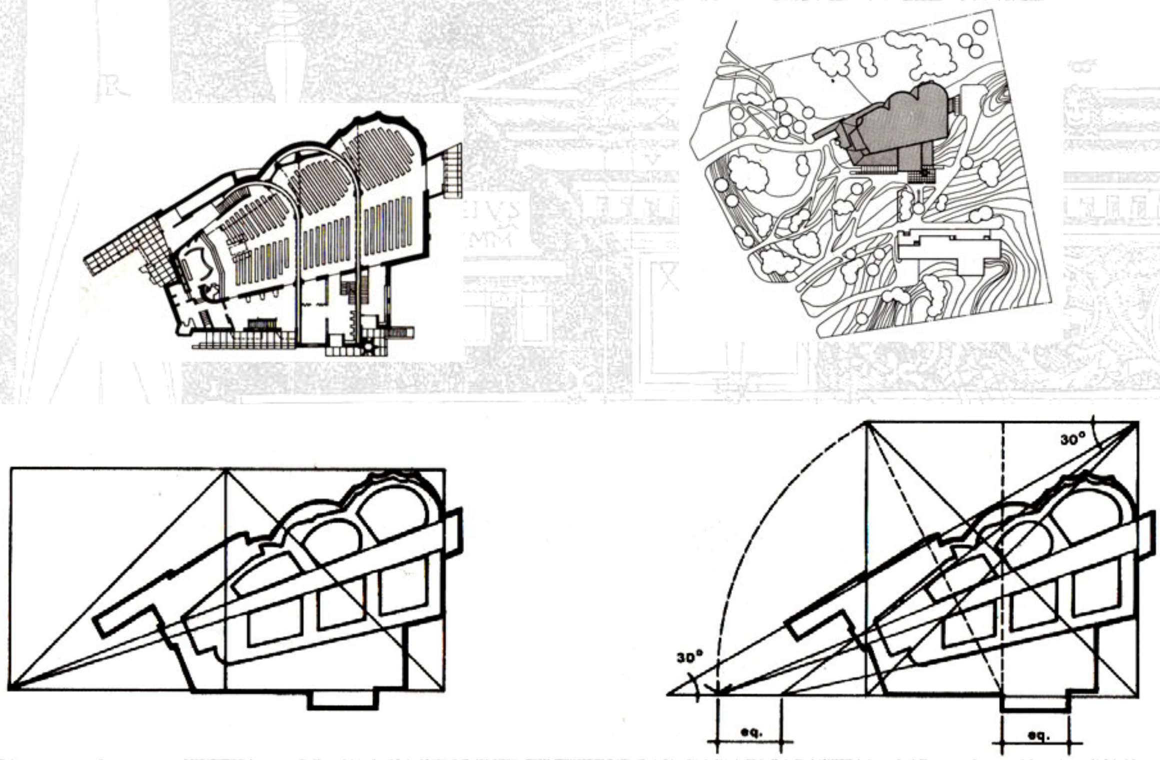


Figure 11: Vouksenniska Church

The usefulness of the different methods varies depending upon the stage of the project.

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(WHOLE UNIT)	+++++++						
UNIT OF USE	+++++++	++++++++					
AREA METHOD	+++++++	++++++++	++++++++	+++++++			
VOLUME METHOD		+++++++	++++++++	+++++++			
SYSTEMS METHOD			+++++++	++++++++	+++++++		
TRADE BREAKDOWN				+++++++	++++++++		
ENCLOSURE METHOD			++++++++	+++++++			
QUANTITY SURVEY-BASED METHODS							
IN-PLACE			+++++++	++++++++	+++++++		
MATERIAL/LABOR					++++++++	++++++++	+++++++

Figure 12: Cost Estimate Stages

The use of mathematics in architectural design comes through practice, implementation and experience. The process is cumulative (a mathematical term meaning to add upon one another) as the design works proceed.

The initial stage of mathematical applications comes through the programming stage. The architectural program is the initial document that details in writing the spaces and zones within the intended facility regarding use, size, requirements and special details. The size category is that component which adds up to the net or gross area for the facility. The sum of the individual spaces adds up to the total net area for the facility.

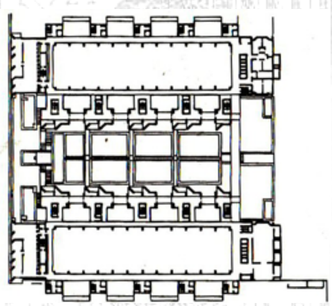


Figure 13: Salk Institute Plan

The calculations continue on using this total amount (T) for the gross calculations. Gross calculations for a facility (addition, renovation or new construction) are meant to include all of the areas not originally identified. Clients may provide a list of required spaces (example: 30 offices of 15 square metres each, or 8 classrooms of 70 square metres each) but the program does not typically provide the required additional areas for circulation, ductwork, and service areas. These additional areas are added in through a percentage calculation.

A brief example of this method follows, using assumed percentages of:

- Circulation - 20%
- Walls - 9%
- Service space - 22%

If we use the first example of offices, the calculation appears as such:

30 (offices) x 15 (SM each) =	450 SM
Service space = 450 SM x 22% =	99 SM
Subtotal -	549 SM
Circulation = 549 SM x 20% =	110 SM
Walls = 549 SM x 9% =	49 SM
Total Area =	708 square metres

We see through this example that the client's offices need only 450 square metres but the total area required amounts to 708 square metres.

The budget for this project can now be established using assumed unit rates for office construction of $700 per square metre. Therefore the equation is:

708 SM area x $700 per SM = $495,600.00

It is at this point where the first check on the budget status can occur. If the client's budget does not equal or exceed the estimated cost, adjustments will be required.

This set of calculations is an extremely abbreviated illustration of a design project. The nature of the construction and design process involves a wide variety of mathematic factors that affect the end costs. These factors include:

.1 Site Cost (SC): cost of land purchase.

.2 Site Development (SD) Cost: cost to clear and ready the site for the upcoming construction.

.3 Building Cost (BC): cost of construction related to the physical building only.

.4 Site Work (SW) Cost: cost to develop the cleared remaining site area for its intended use.

.5 Utility Servicing (US) Cost: cost to provide essential services to the building, including water, sewer lines, gas lines, and electrical service.

.6 Professional Fees (PF) Cost: costs for professional consulting services relative to building and site design.

.7 Permit Costs (PC): cost to municipalities and local authorities to apply for and receive construction/development permits.

.8 Contingency Costs (CC): an allowance, typically a percentage of the building and site costs, to accommodate unforeseen or unknown items during the process.

.9 Equipment Costs (EC): costs for any special equipment supplied by the client that has not been included in the basic building construction cost (BDC).

This combination of items adds up to the basic development cost as follows:

[(SC+SD+BC+SW+US+PC+EC) x CC] x PF = BDC

This equation represents a basic development equation for a simple project. Costs to be considered on any development must include the costs to finance and support the development while it remains under construction. These costs are often referred to as “hidden costs” or the “cost of money” related to development. These costs include:

.10 Carrying Charges ($C): costs resulting from owning, maintaining and keeping the site in order. These costs include taxes, maintenance, security, management fees, insurance, and temporary services. These costs may be incurred even if no construction occurs; therefore even by owning land that sits idle, you pay costs.

.11 Interim Financing Costs ($IF): costs associated with financing the construction process while it is under construction. These costs refer to the interim funding amounts (to pay for ongoing construction), interest charges and financing charges. These costs occur during development, increasing in size as the development proceeds to completion. Once the development is complete, interim financing costs revert to the permanent financing costs.

.12 Permanent Financing Costs ($PF): costs associated with the long-term debt payback schedule. These costs relate to the typical mortgage applied when you purchase a residence.

So the original equation relative to development costs has now added in some ‘hidden costs’ as noted above. These costs add in as follows:

BDC + $C + $IF = New Development Cost (NDC)

The affect of permanent financing costs ($PF) is calculated once the new development cost (NDC) is fixed. Financing, carrying charges, fees and contingency allowances are typically calculated as a percentage of the subtotal. In this way, these costs are proportional to the overall scale of the project, but this method remains flexible right up to completion of the project.

This example presents a simple illustration on estimating and project development calculations. Quantity surveying is an integrated profession in the construction industry, responsible for cost analysis on all types of projects. The architect works with the Quantity Surveyor to confirm the methods of estimating, materials used and any special conditions that may apply to the project. The Quantity Surveyor is quite often one of the first team members to scrutinize the design drawings as they create the preliminary budgets.

Once all budget estimates have been calculated, other factors relative to the building design must be considered. Civic municipalities contain allowances within the development bylaws which detail the:

- Maximum amount of site area that can be covered by a building. This requirement is referred to as the ‘floor area ratio’ calculation. The requirement may be expressed as a decimal (0.75) or percentage (75%). In this example, the percentage means that the proposed structure may cover only 75% of the site area with the remaining 25% treated according to the civic bylaws.

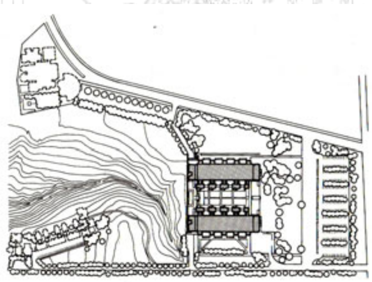


Figure 14: Salk Institute Site Plan

- Area required for landscaping and green space. This area is often expressed as a percentage of the proposed building's gross floor area. This green space may, in some cases, be provided as a component of the building design, or it may be separate landscaped areas.

- Area required for parking, shipping/receiving, accessibility and public access. This area is typically calculated as a percentage of the proposed floor area, depending on the intended use of the building (i.e. warehouse developments don't require as many parking stalls as a restaurant or office building would because of the intended use).

These bylaws, along with the National Building Code, National Fire Code, ASHRAE Standards and Canadian Electrical Code provide guidelines and regulations relative to architectural design and construction based solely on mathematic applications of data. During the process of design, an image or solution has not been finalized; therefore the authorities have no way to advise of the potential impact that the codes and bylaws have on the solution. It would be a waste of time to design a facility that covers 95% of the site, only to find out later that only 60% coverage is allowed.

The requirements of bylaws and codes have been calculated based on years of research on a national level.

The initial way that these articles apply to architectural design is through the mathematic application of area (square metres) which is then used to calculate everything from type of construction allowed (combustible, non-combustible, combinations) to the occupant load (number of persons allowed within the space) to exits required, stair widths, washroom facilities, etc.

The use of mathematics is the means through which the design is defined at the earliest stages. The known or assumed size of the facility will lead the way in estimating, preliminary planning, site usage, code applications and access requirements. Much of the design criteria required to complete initial functional sketches comes through the use of mathematics.

Budget types commonly used in the construction and design industry can be identified according to their classification (as defined by the Government of Canada). The four classifications are:

.1 Class A: highly refined, quantity survey method, elemental breakdown. This estimate is considered very detailed and carries a margin of error of 5 to 10% overall.

.2 Class B: very refined, systems or trade breakdown method, typically completed at the end of the design development stage when the extent and quality of all finishes in every area are known. This estimate is considered to be reasonably detailed and carries a margin of error between 15 to 22% overall.

.3 Class C: not refined, enclosure or volume estimating method, typically completed at the end of the preliminary design stage when basic design relative to building scale, size and quality has been determined. This estimate is considered to be an approximation and carries a margin of error between 22 to 30% overall.

.4 Class D: unrefined, global, ‘ballpark’ type, whole unit, unit of use, or initial area method, typically maintained from initial project stages through the preliminary design stage. This estimate is considered to be very general and carries a margin of error between 30 to 40% overall.

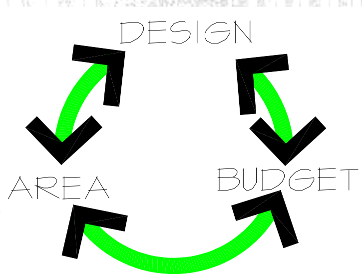
It is demonstrated through the classifications that more refined and precise estimates occur as the design process carries through to construction. The mathematical applications related to the classifications also grow in complexity and detail with each successive stage.

The unit rates of costs for buildings are often referred to as “cost per square metre”. This rate is written as a ratio of cost amount / square metre, where square metre is assumed to equal one unit. Therefore a ratio of \$100/sm means that it will cost \$100.00 to build one square metre of that facility.

The use of fractions in this manner is repeated throughout this section. In every case, the diagonal line and denominator denote “per 1 unit of the item shown” (i.e. $\$100/M^3$ means it will cost one hundred dollars to build one cubic metre (volume method) of the intended facility).

The application (add, subtract, multiply and divide) rules for construction mathematics are the same as those applied to standard fractions. In every case, there must be a common denominator for the equation to be valid.

The initial purpose of mathematics is to act as a performance tool towards understanding the scale, scope and magnitude of a project. The basic requirements and planning parameters of a problem are defined through the use of mathematical calculations and applications. The process is circular, relating reality (areas and budgets) to theory (design rationale) that continues throughout the design stages.



MATHEMATICAL BUILDING ANALYSIS

Mathematics applied to building analysis can supply the information needed to make critical decisions at the outset of a project. The basic thrust of financial analysis is to provide the maximum benefit of design, materials and construction at the most economical cost ratio. This task is not an easy one.

The analysis of design takes on many forms such as:

- materials analysis
- value engineering
- life cycle costing
- project delivery methods

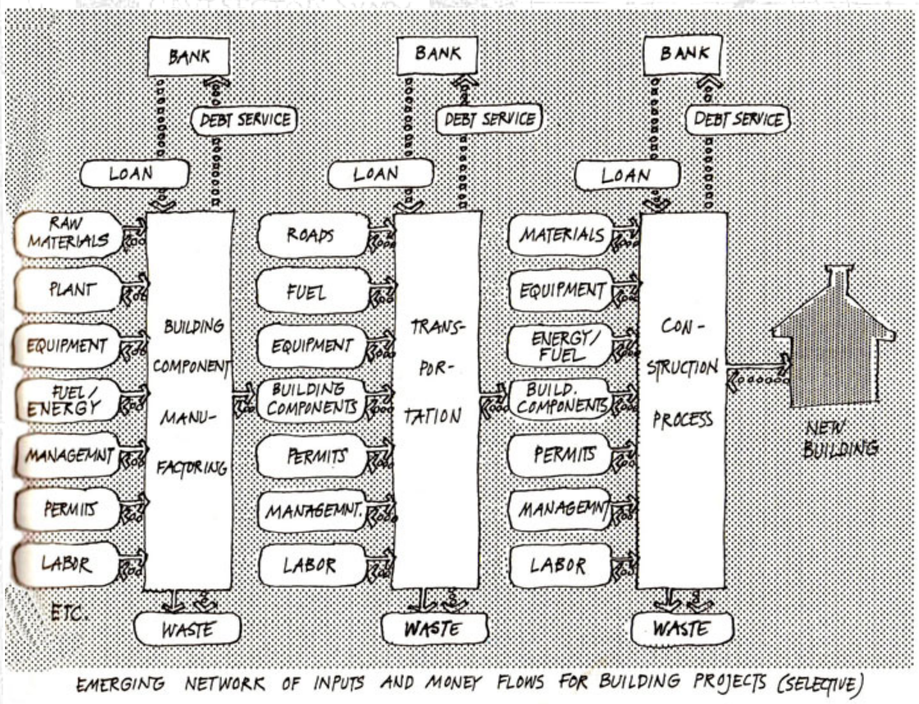


Figure 15: Cash Flow Pattern

Materials Analysis

Materials analysis relates to the decisions of construction materials versus costs. The type of materials may be pre-determined by the client or project (addition to existing building providing the first clue) but there may still be discussion. To cite the earlier example of brick masonry versus stucco as an exterior finish, as discussed in Section 2.0, Science of Buildings –

- brick is expensive (materials and support required), more durable, and provides a distinct appearance;
- stucco is more economical, less durable (subject to more maintenance over time) and can be textured for appearance.

This basic example would be used in the cost estimating stage to illustrate the additional cost for masonry, allowing the client the opportunity to make an informed choice. This type of comparison is applicable to every possible system within the proposed facility (wall types, roofs, ceilings, floor systems, finishes, structure, mechanical and electrical systems).

Value Engineering:

Value engineering focuses on two aspects of architectural design – function and aesthetic. This concept is very similar in method to the Materials Analysis method.

Value engineering refers to analysis of the “function” of specific systems within the proposed design solution and calculates the alternative method costs, comparing them to the original solution method. An example of this is seen in the structural systems of a design – whether it is more economical to use a steel frame, independent of the wall systems, or to use a load-bearing wall system. Value engineering also considers the “function” of aesthetics, as noted in the example of masonry versus stucco within the materials analysis estimate type.

Life Cycle Cost Analysis

Life cycle cost analysis is another method, similar to 'Value Engineering', where proposed solutions are reviewed during the design process. The purpose of Life Cycle Costing (LCC) is to compare different solutions for the same process. This method may be used primarily for the working systems of a facility (heating, plumbing, ventilation/air conditioning) as it carries a component of servicing and replacement costs. New systems may provide better building capabilities, however they may be more expensive to install and maintain than standard, conventional systems. If this indeed proved to be the case, then the client would have spent funds unnecessarily.

Life Cycle Costing calculates the estimated life span (cycle) and serviceability of a system, including estimated operating and maintenance costs, installation or replacement charges and the costs of providing utilities to the system. This method involves reviewing the anticipated use of the facility over the long term in order to provide an accurate comparison. The Life Cycle Cost projections are integrated into the cost estimates to provide opportunities for review of a system's impact on the project.

Other methods of analysis exist, all designed to provide information regarding systems type, performance, costs (short and long-term) and overall budget impact. These analysis methods include 'Rate-of-Return' and 'Cost-Benefit Analysis'. Each method carries a risk of some magnitude, depending on the complexity and scale of unknown factors.

Project Delivery Method

The project delivery method reviews the best possible process to be followed in providing service and eventually a product for the client. Many variations of project delivery methods exist, including:

(1) Design / Bid / Build – The architect works with the client to design a solution, complete construction documents, request bids from contractors, and construct the building with the client's selected contractor. This method is the typical method applied in the industry.

(2) Design / Build – A contractor is selected at the outset of the process to act as a member of the design team, providing input, estimates and professional advice during the design process. The contractor will then take the completed documents and carry out the actual construction work.

(3) Project Management – a system where an independent consultant or contractor will provide expertise in administering the process of design and construction. This position often works in consultation with the architect. A separate contractor is hired in this system, typically using a bid system method.

(4) Construction Management – this method hires the contractor to work with the design team, although not necessarily at the start of the process. This system is similar to Design/Build except for the manner through which the pricing and contracts are received on the project.

The reasons to select one type of project delivery method vary, however time factors and economic influences play a large role in this decision process. A client will often desire to have construction started prior to the length of time required to fully complete a set of contract documents. This scenario may arise due to the required completion date or the fact that the client is paying to maintain a vacant or derelict site, losing money. This method is known as “fast-tracking” a project, starting on foundations at the earliest possible time, likely prior to completing the interior layouts.

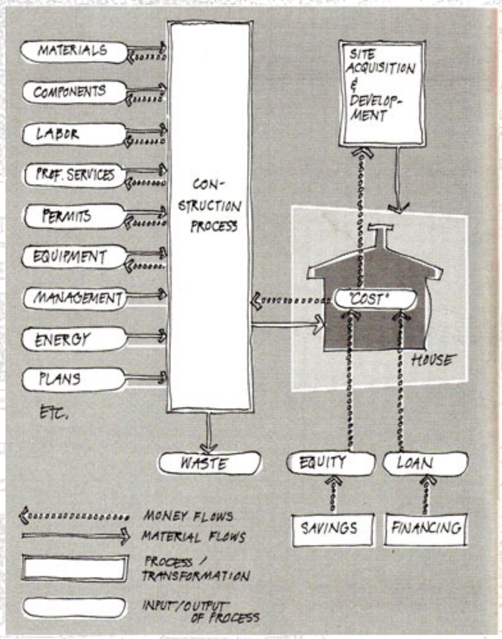


Figure 16: Material/Finance Flow Chart

Risks accompany all of these project delivery types. These risks exist in the example of Design/Bid/Build if the bids received exceed the client’s budget. This situation should not transpire if the architect’s budget is accurate but it does occasionally occur. This situation requires a decision from the client: whether to delay the project and go through the process of reducing the project scope or whether to commit additional funds to the project and proceed at the higher amount.

Risks in the other delivery types exist also. These risks all involve the use of mathematics as contingency amounts, allowances and percentages in order to allocate funds suitable to offset the potential risks. It remains a challenge to select the best design solution with the most economical, time-effective delivery method.

Design Area Calculations:

As a working example, the design areas for this thesis are illustrated here. Design areas were calculated for the program requirements of this thesis based on the intended student population, number of instructors and the allowable area for the function of the space. These calculations were incorporated into a spreadsheet that developed the overall gross area for the intended design solution. A copy of the summary sheet follows.

		Staffing/Students	**Area**	
Item.	**Area Name**	**Units**	**Unit Rate**	**Subtotal**
1.0	**Instructional Area**			
	Lecture Area			888.75
	Lab Area			804.33
				1693.08
2.0	**Resource Area**			
	General Resource Area	319	0.5	159.60
	Seminar / Computer	9	5	45
	Resource Administrator	9	7	63.00
	Media Storage	268	15%	40.14
				307.74
3.0	**Administration**			
3.1	Administrator's Office	1	14.00	14.00
3.2	General Office	2	12.00	24.00
3.3	Workroom	319	0.20	63.84
3.4	Visiting Lecturer	1	12.00	12.00
3.5	Staff Room	319	0.20	63.84
3.6	Staff Lockers	32	0.44	14.04
3.7	Staff Washrooms	23	4.40	13.49
3.8	General Storage	319	0.15	47.88
				253.10
4.0	**Building Support Services**			
4.1	Student Washrooms	21	3.00	63.84
4.2	Maintenance Areas		10%	201.00
4.3	Building Service Areas		5%	110.55
4.4	Student Commons		20%	338.62
				714.01
5.0	**AREA TOTAL**			**2967.93**
6.0	**Circulation**		20%	**593.59**
7.0	**Wall Allowance**		9%	**320.54**
8.0	**GRAND TOTAL**			**3882.05**

This spreadsheet was developed using a student population of 320 students which provides an area result of 3882.05 square metres. The affect of adding an additional class of twenty-five (25) students will have a major impact on the facility, increasing the total area to become 4246.65 square metres; an increase of 364.60 square metres. If we apply an estimated construction cost of $1600/M^2$, then one additional class will add $583,357.00 to the initial construction budget.

In this brief example, the illustration shows that one student in the planning stage will add $23,000 to the cost of construction. This construction cost is increased by adding in professional and development costs as well as the consideration of long-term operating and maintenance costs. It is through the use of mathematics that issues and questions such as this illustration are discussed to resolve the design parameters at the outset of a project. A design solution is often discussed and resolved numerically as part of the architectural design process during the initial sketching of forms and ideas.

The second part of this curriculum section discusses the use of geometry, proportional design and applications of the geometric principles (specifically the "Golden Mean") in architectural design.

GEOMETRY

Geometry is used in every aspect of architectural design. The basic forms of circle, square and triangle can be found throughout architectural design solutions. Squares may be elongated in one or two directions to create rectangles, triangles may be offset on the apex and circles may be altered to form arcs or ellipses. In every case, the basis of these three geometric shapes can be found (circle, square, triangle).

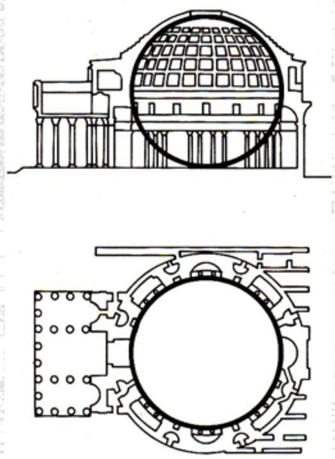


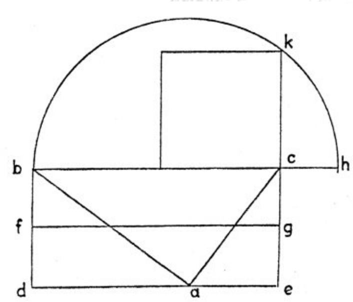
Figure 17: The Pantheon, Rome

The use of geometry satisfies a crucial relationship between mankind and our environment. As we are one species that must build our own environment, the physical and perceptual relationship we maintain with the world around us is determined by our sense of what is "good" or "bad".

A review of design principles is contained in the 'Design Elements' section of this curriculum. This section of Mathematics will focus on the basis of geometry, found case studies and the design intent implicated in its use.

Geometric patterns are all around us. The principal three found in architectural design as noted herein are the:

- Circle: translates into an ellipse, arc, parabola or contiguous sine wave. This item represents the "fluid motion" of architectural design solutions.
- Square: translates into a rectangle or grid system. This item represents the "fixed" element of architectural design solutions. Weaknesses can be perceived on the square as it has corners, or connections, that "change" the direction of the line. A circle as a pure form does not present any "weak" sides.
- Triangle: translates into modified squares or rectangles, creates what can be perceived as "slope" within design, leading the eye or participant in a specified direction. This item provides a sense of "movement" as a rectilinear pattern; straight, not fluid as a circle. A triangle in architectural design creates focus and direction, although like a rectangle it does present "weak" sides and even dominant lines within itself.



We see these items as primary geometric shapes though it is their relationship to mankind itself that creates the perceptual importance of these items.

The geometry of mankind develops proportional relationships relative to the human figure. These relationships form the idea of "good" or proper elements of our perception, basing what we see in the world around us within the context of what we "feel" regarding ourselves.

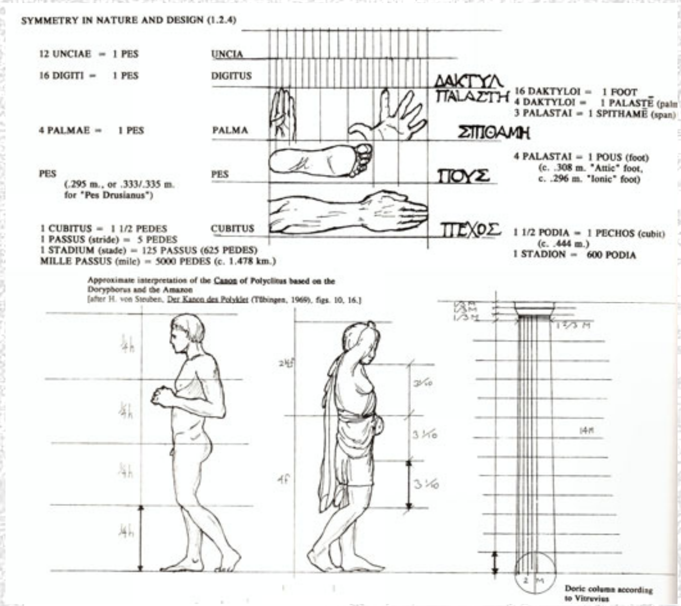


Figure 18: Human Proportions (Vitruvius)

The use of the basic forms noted can be found in the following ways.

(1) Circle: The distance from fingertip to fingertip of horizontally outstretched arms will be equal to a person's height; therefore a circle drawn using the soles of the feet and crown of the head as diameter will have the same diameter as the outstretched arms. This relationship creates a "spherical" sense of our space, knowledge of the sphere and circle is thereby inherent in our senses relative to our environment. This relationship is maintained with the body forming an "X", having the outstretched hands and soles of the feet all touching upon the same circle.

(2) Square: Our perception of the square is based on the same rationale as the circle. Our height equals our reach which creates two equal dimensions of the square.

(3) Triangle: The triangle completes the basic shapes relative to mankind by the union of straight lines within the body. These relationships can be found in a multitude of ways and combinations, including the ways our limbs move based on two fixed (i.e. elbow and shoulder) and one flexible (hand) connection. This combination of three points (two fixed and one flexible) is repeated in the legs (hip, knee and foot connection) and in the body (hips, shoulders and head connection).

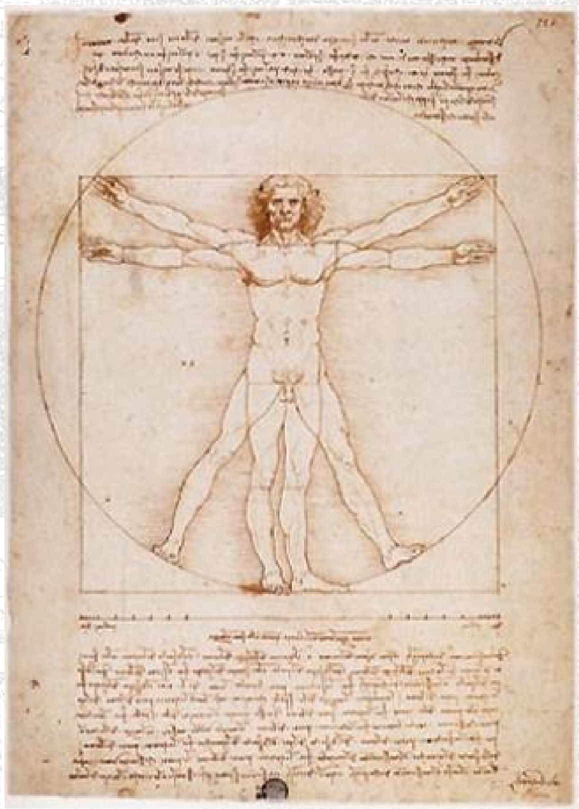


Figure 19: The Proportions of Man
(Leonardo Da Vinci based on Vitruvius)

We see these geometric shapes in the world around us, starting at a young age, forming our preconceptions of form and functions. These basic forms create a sense of clarity and completeness in our perception of design; a room's width equaling its length, is simple and easy to take in and subsequently accept. This perception is thrown off in cases of long, narrow spaces (corridors) as the shape does not fit within our cognitive comfort zone.

The use of squares and circles is evident throughout our environment, although not every building is made up of the two geometric shapes. Abstractions and interpretations of these shapes have occurred in order to provide sufficient area for the desired use. These abstractions are how proportional geometry is used in architectural design.

Proportional geometry is the process of enlarging or decreasing the basic square to a new rectangular shape in order to suit the specific problematic requirements. The essence of these proportional shapes reflects a relationship in keeping with our perception of what is "right" or in the case of architecture, aesthetically pleasing.

A mathematical formula has been developed through history which is called the "Golden Ratio", or in some cases, the "Golden Mean". ("Mean" in this case relates to a 'standard' rule of application, not an emotion).

Architectural designers have searched for a consistent, hard rule with regard to design as far back as ancient Greece and Egypt. Early Greek philosophers such as Pythagoras (geometric rules of Pythagoras' theorem) found that objects which were proportioned according to the Golden Mean (1:1.6) consistently appeared more attractive and pleasing than others. This ratio produces a rectangle/square combination which is termed the 'Golden Rectangle'.

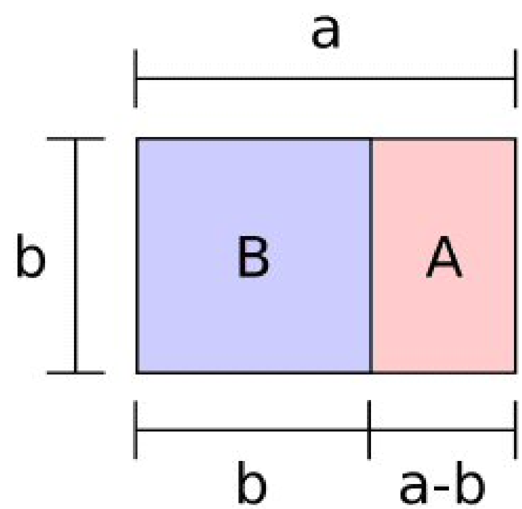


Figure 20: The Golden Mean

A Golden Rectangle is a geometric shape whose side dimensions equate to the ratio of 1:1.68033. When the rectangle is sectioned off into a square (relating to the initial discussion of 'perfect' shapes), the remaining area creates another Golden Rectangle whose sides maintain the ratio noted.

This process can be applied infinitely to enlarge or break down the original shape. Starting with a square (or circle with an inscribed or circumscribed square), it becomes a geometric progression to create Golden Rectangles, proportional square and circles, thus enlarging the pattern to suit any nature of design.

This method may be used for almost any facet of architectural design. It is true, though, that architectural design is not merely squares and rectangles since there is line and motion applied through design philosophy.

The Golden Rectangle, as divided down, provides us with the logarithmic spiral, otherwise known as the Golden Spiral. This arcing spiral occurs naturally in the world around us; seashells, sunflowers, etc. It is an interesting coincidence that this spiral suits the nature of the Golden Mean, fitting the rectangular dimensions so perfectly.

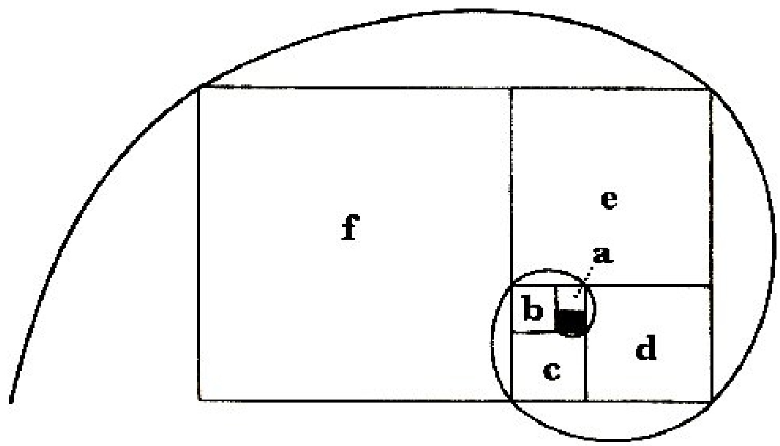


Figure 21: The Golden Spiral

The use of these proportions and dimensions architecturally can be found in examples throughout history. The early design methods did not have the availability of computers that could calculate incredible numeric sequences in milliseconds. Historically, design relied much more on the geometric planning relative to a structure to determine its aesthetic. The use of geometry requires only one known dimension (the side of a square or triangle or the diameter/radius of a circle). All proportions, heights, and widths and even thickness of elements can be derived knowing this one dimension.

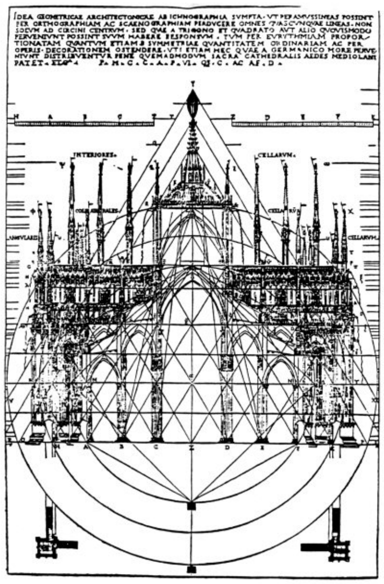


Figure 22: Milan Cathedral

The use of mathematics in architectural design has been noted throughout history. Mathematicians even played an integral role during construction of the Milan Cathedral, as noted in this excerpt from the meeting record. The discussion of record related to the height and delineation of the main building façade.

> *"The solution of this dilemma was not in the province of an architect, and a mathematician was summoned. (the question being) whether this church…. ought to be built according to the square (ad quadratum) or the triangle (ad triangulum). It was stated (by the mathematician) that it should rise up to a triangle or to the triangular figure, and not farther.* [2]

This use of mathematics in architectural design continues to this day, except that mathematicians are typically not summoned when a design crisis erupts with the client.

[2] Case Study #4, Didactics

The use of the Golden Rectangle can be illustrated in the way that the façade of the Parthenon on the Acropolis can be detailed out. The overall geometric rectangle breaks down into the gable end, frieze, and column sections.

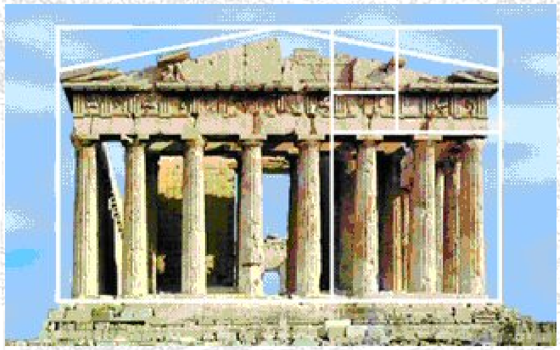


Figure 23: The Parthenon

Geometry can be traced back to the Egyptian period when the pyramids were laid out. The geometry of the pyramids relates to the determination of the side slopes, a clear example of mathematics and geometry. The slope incline is determined by the height of the triangle to the radius of a circle. The circumference of this circle equals the square plan of the pyramid itself. Therefore, knowing one dimension of the square (side) of the footprint would allow the remaining dimensions/shapes to be drawn out without requiring additional measurement. Each of these elements is connected, allowing for a clear proportional development to occur based on fixed elements. Nothing in this method is left unplanned.

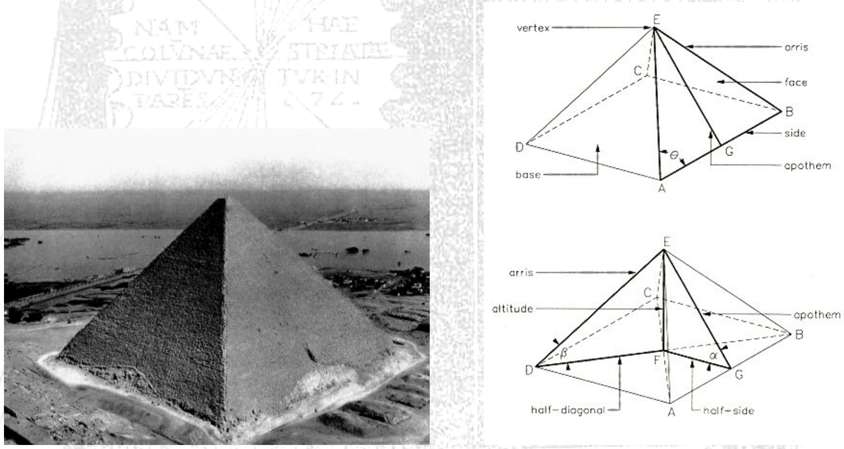


Figure 24: Pyramidal Analysis

The determinant in mathematical calculations is the ratio by which a number must grow or shrink in order to maintain a “proportional” value. This equation has been studied by mathematicians, artists, and architects throughout history, arriving at a sequence formula known as the Fibonacci Series.

The Fibonacci Numeral Series

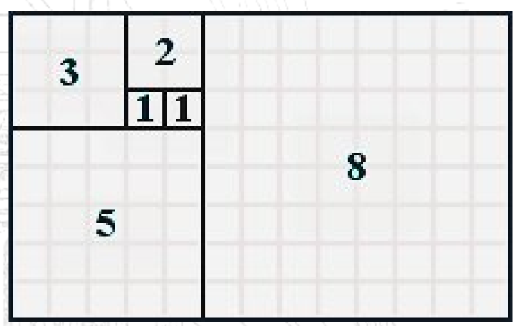
The Fibonacci Series is both complex yet incredibly simple. The simple side relates to the equation “1 + sqrt5 / 2 = a“. The end result for “a” will be 1.618033… This number related to ‘1’ is the Golden Ratio. The Fibonacci Series uses the Golden Ratio as the basis for its mathematical derivation.

The Fibonacci Series starts at 0, increasing by the addition of the previous two numbers to create the next number in the sequence. The formula is:

0 + a = b
a + b = c
b + c = d
c + d = e
d + e = f, and so on for infinity

Mathematically, this equation appears as:

0 + 1 = 1
1 + 1 = 2
1 + 2 = 3
2 + 3 = 5
3 + 5 = 8
5 + 8 = 13, and so on…



The importance of the Fibonacci Series is that the ratio of any one number in the sequence to the next number (higher or lower) is a derivation of the Golden Mean (1.618033…). This calculation is flawless, although the example herein uses whole integers rather than the full number.

The relationship of any two numbers in this series equals the proportional ratio of the Golden Rectangle, which has been deemed or judged as aesthetically pleasing over history. The relationship of the geometry to the mathematic applications can therefore follow either a graphic geometric breakdown related to the Golden Rectangle or a mathematical ratio related to the Fibonacci Series (multiplier of 1.618033…). These ratios represent a modular system.

Le Corbusier created his own graphic of the mathematical modular system, known as "The Modulor". Using proportions of the human form, he achieved the basic dimensions of "a" (113 cm – red series) and "b" (226 cm – blue series). These basic dimensions are multiplied by the Golden Mean to arrive at the required proportions for his architectural designs.

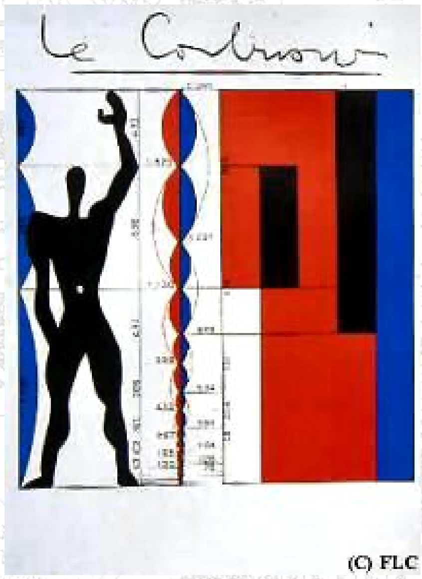


Figure 25: The Modulor

These proportional relationships are detailed within the writings of Vitruvius through the analysis of design and detailing for plan, elevation and section of the various buildings types.

A basic example relates to the designed proportions of Doric symmetry:

> *"The façade of a Doric Temple should be divided along the stylobate (base) into twenty-seven parts if the building is going to be tetrastyle; if hexastyle, into forty-two (parts). One of these parts will be (determined as) the MODULE. Once this module has been decided, all the calculations for the proportions of the whole project may be carried out. "*[3]

This basic planning process according to Vitruvius was based on the initial decision for the temple; if it is to be tetrastyle or hexastyle. Every subsequent decision past that point was based on the established ratios for the building type. An example of this breakdown using "M" as the module is:

- the diameter of a column = 2 M
- the column height including capital = 14 M
- the column capital height = 1 M
- the column capital width = 2-1/6 M
- the epistyle height including taenia & guttae = 1 M
- the taenis height = 1/2 M
- the guttae (gutter) height = 1/6 M
- the triglyphe including metopes = 1-1/2 M high x 1 M wide

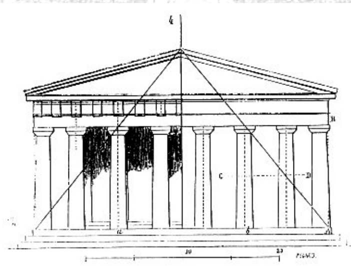


Figure 26: The Parthenon, Athens

The overall plan division was also prescribed according to the initial division such that the width of the interior temple was to be equal to one-half its length. The cella (interior room) shall be longer by one-fourth than its width.

[3] Vitruvius, The Ten Books of Architecture, P. 57

These breakdowns were applied throughout the realm of architecture during that time. These principles produced structures that have withstood the test of time and analysis, being regarded today as true examples of design and proportion. This proportion remains relevant due to the fact that its basis for discovery was the human figure.

When citizens of Ionia first built a temple to Apollo, they used the measurement of a man's foot; that measurement being equal to 1/6 his height. Therefore, once they decided on the diameter of the column base, they carried the height to six times the diameter. In this way, the Doric Column came to exhibit the proportion, soundness and attractiveness of the male figure.

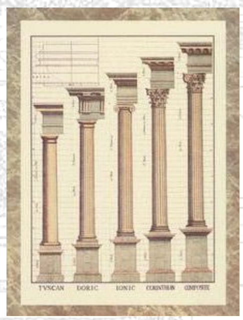


Figure 27: The Five Orders of Proportion

Upon building a temple to honour Diana, they chose the female figure; using the slenderness of the footprint multiplied by seven times to equal the height. In this manner, the Ionians created the symbolism of male form and proportion in the Doric Column, and the female form in the Ionic Column.[4]

Later generations further refined these proportions and ratios with the subsequent adjustments of the Doric height becoming seven times its diameter and the Ionic column being nine times the diameter.

[4] Vitruvius, The Ten Books of Architecture, P. 54

The aesthetic use of the Golden Mean remains as much in debate as it does in universal acceptance. This ratio or geometric system appears to be applicable primarily through cultural and historical interpretation, specifically to Western cultures. Islamic and East Indian cultures use similar ratio derivations such as 1:2 of Islamic forms or the 3-7-22 rhythm of East India.

The use of the Golden Rule is not required within our culture, although its proportional relationship has proven over time to produce spaces that appease our perceptual senses.

The Golden Mean relates to mathematical proportion, found throughout nature, even within ourselves. Geometric analysis is a means of architectural resolution that uses basic theory relative to our perceptual senses as to what is to be considered “right” in its breakdown and appearance. The use of geometry within architectural design may be found in an application as simple as the theory of right angles.

The use of the right angle provides a method for breaking an overall plan down into sections that inadvertently make sense to the viewer. The primary facet of this theory is the initial angle of a diagonal line across the outside enclosure.

The ‘enclosure’ referenced relates to the outer perceived limits of the plan, elevation or section. Regardless of the application, the theory works by using the angle of intersection to the diagonal as the geometric basis for all other elements within the enclosure.

Right angle theory provides proportional relationships within each element that are equal to the proportion of the whole. In this way, the eye perceives a proper sizing of the parts to the whole. If the relationship is equal to the Golden Mean, then all subsequent relationships will be also. If the relationship is of an alternate ratio (i.e. 1:3.5), then all subsequent ratios will equal that of the main. In this way, the proportion of parts to the whole is maintained.

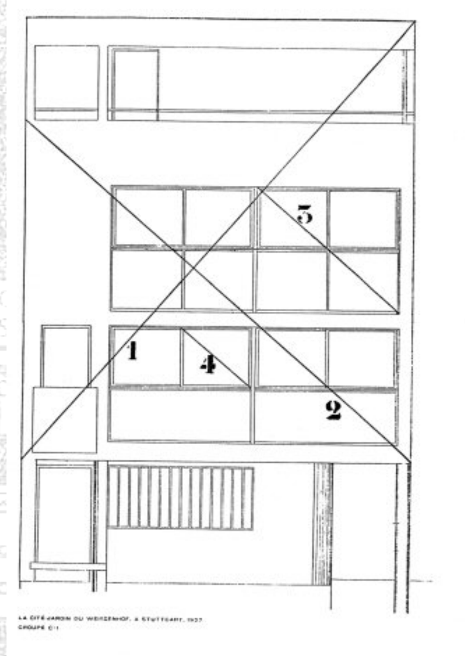


Figure 28: Right Angle Analysis, Le Corbusier

The theory of right angles provides one identifiable aspect in the relationship of parts to the whole. The mathematical ratio of the whole is reversed in the ratio of the parts. For example, if the ratio of width to height of a building façade is 4:1, then the right angle delineation will provide a ratio within the parts of 1:4. The long dimension becomes perpendicular to itself; the short dimension also. This aspect provides for our perceptional senses to respect the relationships and ratios without making the design appear repetitive or forced.

There remain many opportunities for the use of geometry within the practice of architectural design. The use of four-square or nine-square planning, grids which then translate into the elevation and sectional grid for volume, can be implemented.

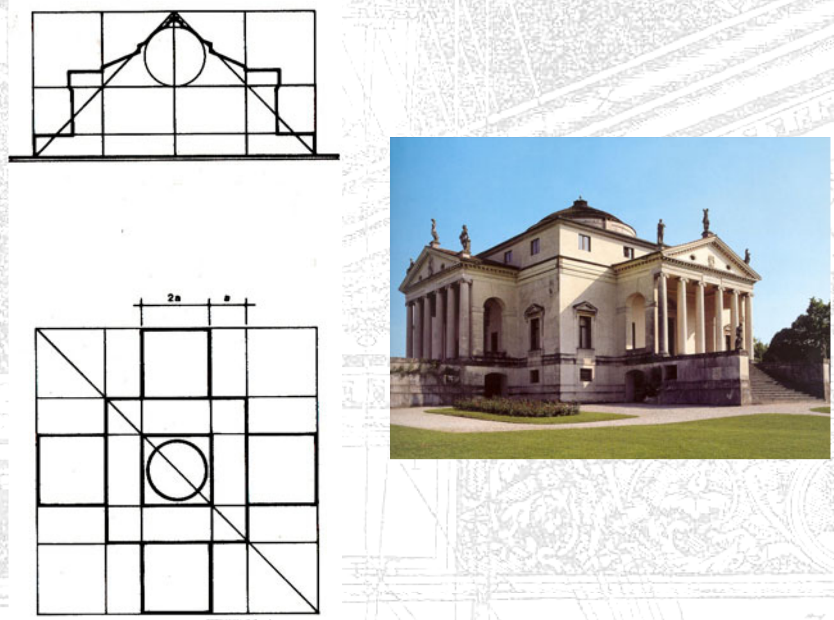


Figure 29: La Rotunda Villa, Palladio

Geometry is used throughout architectural design to complete the relationships within plan, elevation and section. Repetitive geometry is also implemented in a successful design solution to complete the design relationships.

The method used is at the discretion of the architect. It is important to maintain the method throughout the design solution in order to create the coherence needed to be able to "read" the building form in two and three dimensions.

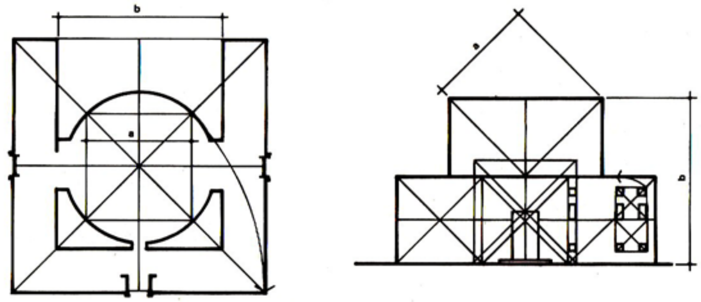


Figure 30: Stockholm Civic Library Geometry

Geometric method stems from the original design parti created at the beginning of the design process. It is through the parti that the design rationale for the solution is predicted and it is through use of the parti geometry in the design solution that the rationale is proven.

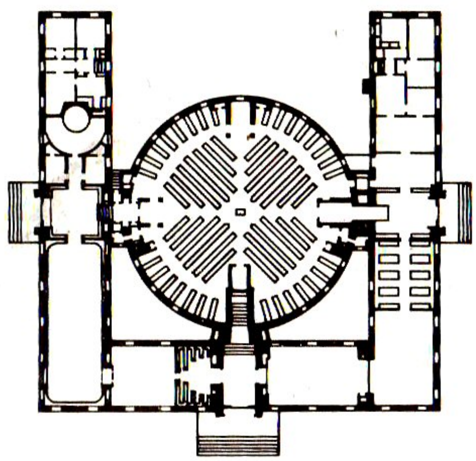


Figure 31: Stockholm Civic Library Plan

Geometry can be used in many ways within architectural design, outside of the principles of the parti solution. Frank Lloyd Wright created stained glass windows to frame the views of the exterior in colour. His forms were simple geometric patterns (squares, rectangles and triangles) to create the natural forms in a geometric configuration.

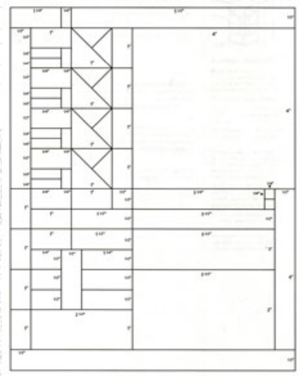
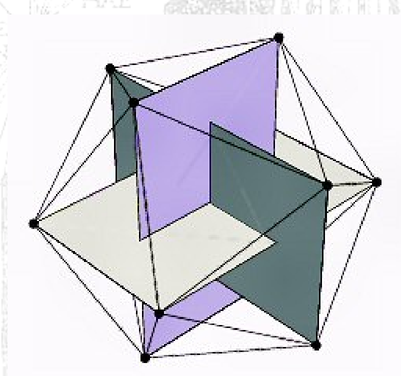


Figure 32: Corner Section, Wright Window Design

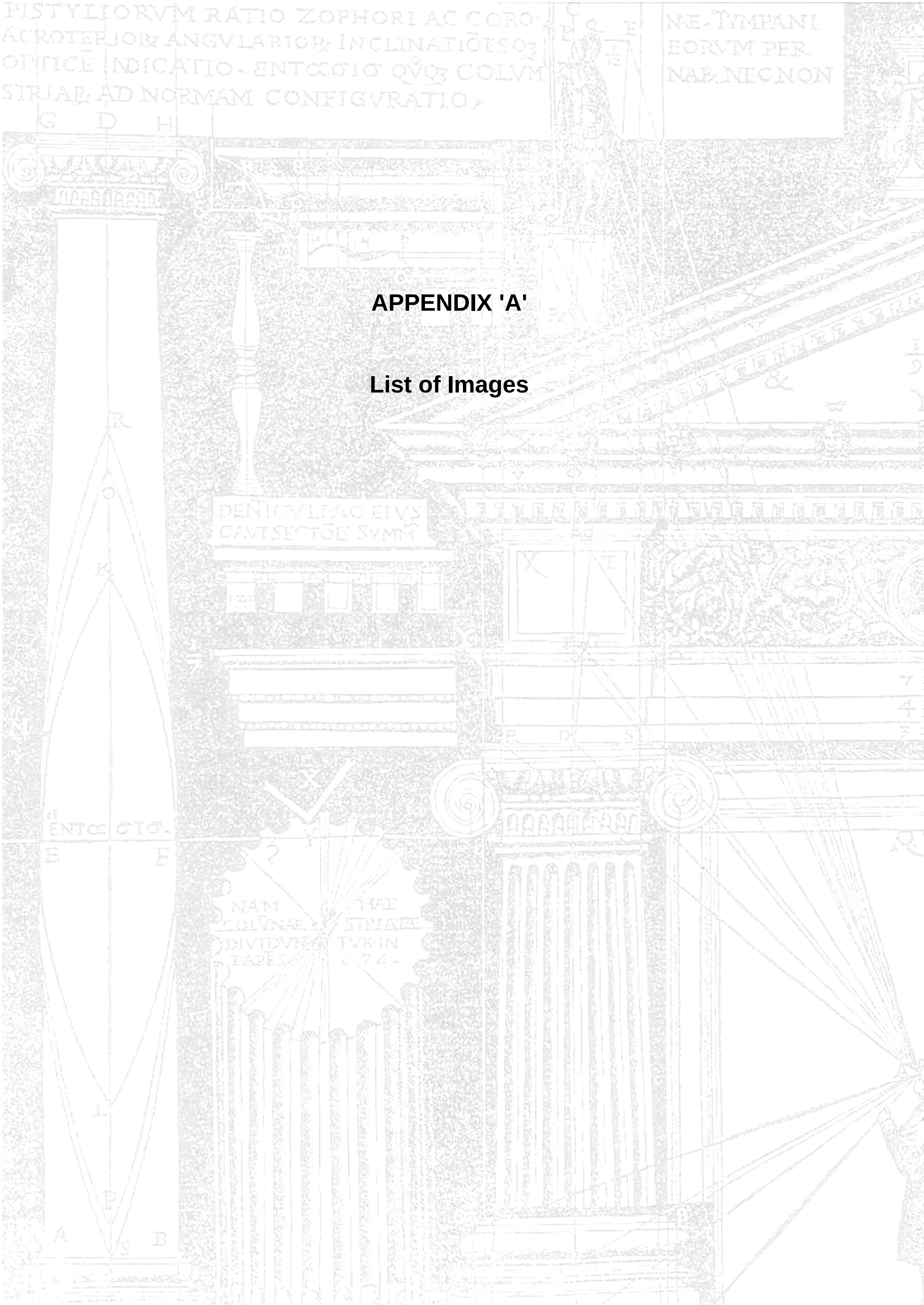
Each of the windows Wright designed was planned on a grid system with the organizational methodology of symmetry and repetition. These windows represent one use of geometry to enhance the overall aesthetic of architectural design. Additional examples can be found within the Art Deco stream of design present during the early 19th Century.

Mathematics and geometry form an integral part of the basis of architectural design. Mathematics provides the initial knowledge relative to the potential solution (scale, requirements, scope and budget) while geometry provides the rationale through which the design solution may be generated. These two aspects and contributions to the overall design process are critical in developing a methodology towards success.



NEW TEXT DEFINITIONS:

{A listing of new architectural definitions provided by this component}



APPENDIX 'A'

List of Images

Reference tags:

- A : Image by Author
- B : Building Economics for Architects
- G : Internet Image
- N : Nexus Network Journal
- P : Precedents in Architecture
- V : Ten Books on Architecture

Number	Name	Reference	Page
Cover	Icosahedron	G	---
1	Fibonacci Algorithm	Mario LIvio	---
2	Total Building Cost	B	16
3	Stockholm Civic Library Parti and Geometry	P	23
4	Circular Division	N	11
5	Paris Opera House	Architecture: Pre-History – Post-Modernism	450
6	Budget Breakdown	B	9
7	Human Figure Ratio	Mario LIvio	---
8	Cost Estimate Stages	B	24
9	Gross Floor Area Outline	A	---
10	Net Floor Area Outline	A	---
----	Cost Estimating Diagrams	B	17
11	Vouksenniska Church	P	10-11
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30	Stockholm Civic Library Geometry	P	23
31	Stockholm Civic Library Plan	P	22
32	Corner Section, Wright Window	G	---
	Icosahedron	G	---

APPENDIX 'B'

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